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**Green University of Bangladesh**

**Department of Computer Science and Engineering (CSE)**

**Faculty of Sciences and Engineering**

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**Assignment - 02**

**Course Code: CSE 101**

**Course Title: Discrete Mathematics**

**Section: DJ**

**Assignment Topic: Counting**

**Student Details**

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| **Assignment Status**  **Marks: ………………………………… Signature:.....................**  **Comments:.............................................. Date:..............................** |

This assignment has the solution of the Topic: Counting

The solution to the questions of

Page 396 and Q: 21, 24

Page: 405 and Q 1, 9, 15, 18

Page: 413 and Q 14, 18, 20, 21, 26, 31

1. **Problems of Page – 396:**

* **21.** 100 and 50 is divisible by either 7, whether the ranges are meant to be inclusive or Exclusive.

**a).**

There are 100/7 = 14.28 ~ 14 integers less than 100 that are divisible by 7,

and 50/7 = 7.14 ~ 7 of them are less than 50 as well.

This leaves 14 - 7 = 7 numbers between 50 and 100 that are divisible by 7.

They are 56, 63, 70, 77, 84, 91 , and 98.

**b).**

There are 100/11 = 9.09 ~ 9 integers less than 100 that are divisible by 11,

and 50/11 = 4.54 ~ 4 of them are less than 50 as well.

This leaves 9 - 4 = 5 numbers between 50 and 100 that are divisible by 11.

They are 55, 66, 77, 88, and 99.

**c).**

A number is divisible by both 7 and 11 if and only if it is divisible by their least common multiple

which is 77.

There is only one such number between 50 and 100, which is 77.

We could also work this out as we did in the previous parts:

100/77 - 50/77

= 1 – 0

= 1

Note: The intersection of the sets we found in the previous two parts is precisely what we are looking for here.

* 24.

Soln:

This problem deals with the set of positive integers between 1000 and 9999, inclusive. Note that there are

exactly 9999 - 1000 + 1 = 9000 such numbers. A second way to see this is to note that to specify a four-digit

number, we need to choose the first digit to be nonzero (which can be done in 9 ways) and then the second

and third digits (which can each be done in 10 ways) and fourth digits (which can each be done in 10 ways) for a total of

9 · 10 · 10 · 10 = 9000 ways, by the product rule. A third way to see this there are 9999- 999 = 9000 numbers in the desired range.

**a).** Objective is to find the positive number between 1000 and 999 inclusive divisible by 9

Every ninth number- 9, 18, and so on-is divisible by 9. Therefore, the number of positive integers less

than or equal to *n* and divisible by 9 is ***n/9*** (the floor function)

So, we find that there are 9999/9 = 1111 multiples of 9 not exceeding 9999, of which

999/9 = 111 do not exceed 999. Therefore, there are exactly 1111 - 111 = 1000 numbers in the desired range

divisible by 9.

b). Objective is to find the positive number between 1000 and 999 inclusive are even

This is similar to part (a), with 9 replaced by 2.

The integers between 1000 and 9999 both inclusive are 9000

Here we see that there are 9000/2 = 4500 even numbers.

Or,

The number in end position must be 0, 2 ,4, 6, 8 for even numbers. There are 5 choices.

The number in the first position must be any number from 1 to 9 and there are 9 choices

For the second and third number there are 10 choices and for the last position we mentioned above 5 choices are possible.

So Total number of choices of getting even number is

= 9 \* 10 \* 10 \* 5

= 4500

Therefore, there are 4500 even numbers between 1000 and 9999 inclusive.

c). Objective is find the positive integers between 1000 and 9999 inclusive have distinct

digits.

In the thousand’s place, there are 9 numbers from 1 – 9.

The hundred’s place if filled with 9 numbers because the number have distinct digits.

The next position have 8 digits from from 0-9 (except thousand’s place and hundred’s place digits).

The one’s palce is filled with 7 digit (except 1000’s, 100’s, and tens place digits)

So, the positive integers between 1000 and 9999 inclusive gave distinct digits are:

= 9\*9\*8\*7

= 4536

d). Objective is to find the positive integers between 1000 and 9999 inclusive are not divisible

by 3.

Number of integers divisible by 3 is [Foor Funciton] = 9000/3 = 3000

So, the number of integers that are not divisible by 3 is 9000 – 3000 = 6000 (Ans.)

e). Objective is to find the number of integers divisible by 5 or 7 between 1000 and 9999 inclusive.

Let A1 , A2 the set of integers which are divisible by 5, 7 respectivly

integers divisible by 6:

[A1] = ┘

= 1800

[A2] =

= 1286

f). Integers divisible by both 5, 7 are LCM (5, 7) = 35

| A1 U A2 | = | A1 + A2 | - | A1 | ᴖ |A2 |

= 1800 + 1286 – 257

= 2829

Therefore, the number of integers divisible by 5 or 7 between 1000 and 9999 is 2829.

g). Objective is to find the positive integers that are between 1000 and 9999 are not divisible by either 5 or 7.

The integers which are not divisible by either 5 or 7 are

Total number of integers - integers divisible by 5 or 7

= 9000 – 2829

= 6171.

Hence, the number of integers that are not divisible by either 5 or 7 is 6171.

h). Objective is to find the integers between 1000 and 9999 inclusive that are divisible by 5 but not by 7.

The number of integers which are divisible by 5 but not 7 is:

It is nothing but an (AB) where A is integers divisible by 5, B is integers divisible by 7

We know (AB) U (AB) = A and AB, AB are disjoint sets,

So,

|AB| = |A| - |A ∩ B|

= 1800 – 257

= 1543

Hence, the number of integers between 1000 and 9999 inclusive that are divisible by 5 but not by 7 is 1543.

1. **Problems of page – 405**

* 1.

There are **six classes** and there are **five days** on which classes may meet (Sunday, Monday, Tuesday, Wednesday and Thursday). **Six classes are six Pigeons and Five days are five Pigeonhole**.

Each class must meet on a day (**each pigeon must occupy a pigeonhole**).

By the pigeonhole principle at least one day must contain at least two classes.

* **3.**

**a)** There are two colors (two pigeonholes). We want to know the least number of pigeons needed

to insure that at least one of the pigeonholes contains two pigeons (two socks).

By the pigeonhole principle the answer is 3.

**If three socks are taken from the drawer**, **at least two must have the same color**. On the other hand two

socks are not enough, because one might be brown and the other black.

**b)** He needs to take out 14 socks in order to insure at least two socks are black. If he does so, then at most 12

of them are brown, so at least two are black. On the other hand, if he removes 13 or fewer socks, then 12 of

them could be brown, and he might not get his pair of black socks. The number of socks did matter here.

* **5.**

There are four possible remainders when an integer is divided by 4. These are 0, 1, 2, or 3(Total 4 pigeonholes here).

Therefore, by the pigeonhole principle at least two of the five given remainders (2 pigeons of total of 5 pigeons)

must the same.

* **15.**

We can apply the pigeonhole principle by grouping the numbers into pairs (subsets) that add up to 7,

which are {1,6}, {2,5}, and {3,4}.

If we select four numbers from the set {1,2,3,4,5,6}, then at least two of

them must fall within the same subset, since there are only three subsets. Two numbers in the same subset

are the desired pair that add up to 7. We also need to point out that choosing three numbers is not enough,

since we could choose {l, 2, 3}, and no pair of them add up to more than 5.

* 18.

**a)**

If this statement were not true, then there would be at most 4 males and 4 females, for a total of at

most 8 students. This contradicts the fact that there are 9 students in the class.

So, at least there are 5 males or at least there are 5 females in the class.

**b)**

If this statement were not true, then there would be at most 3 male students, at most 6 female students, for a total of at most 8 students. This contradicts the fact that there are 9 students in the class.

1. **Problems of page – 413**

* **14.**

Here, we are not interested about the order in which the two integers are chosen so the method of combination is a good approach.

We have 99 choices for the first number, then 98 for the second number, because the order isn’t a issue here.

For double-counted each possible set of two integers, so we should divide by two to get the number of possible sets.

The Calculation:

C (99,2) = Combination Rule: [C(n, r) = ]

=

=

= 4851 (Ans.)

Therefore, we can choose a set of two positive integers less than 100 in 4851 ways.

* **18.** The possible outcomes of flipping a coin is either head (H) or tail (T)

a).

The number of total possible outcomes when flipping for 8 times.

For each toss, there are two possible outcomes, head and tails

By product rule,

2\*2\*2\*2\*2\*2\*2\*2 = 2^8

= 256

So, the total number of possible outcomes is 256.

b).

The objective is to find the number of possible outcomes that containing at least 3 heads.

The number of outcomes with exactly 3 heads is:

C (8, 3) =

=

=

= 56

Therefore, the number of outcomes with exactly 3 heads is 56.

c).

The objective is to find the number of possible outcomes that contain at least three heads

The number of outcomes with at least 3 heads is:

C (8, 3) + C (8, 4) + C (8, 5) + C (8, 6) + C (8, 7) + C (8, 8)

= +

= +++++

=

= 56+70+56+28+8+1

= 219

Or, we can find it another way like:

(Total number of possible outcomes) – [ (8, 0) + C (8, 1) + C (8, 3)]

= 256 – (1+8+28)

= 219

Therefore, the number of outcomes with at least 3 heads is 219.

d).

The objective is to find the number of possible outcomes that contain same number of heads and tails.

There is only one possible outcome to get the same number of heads and tails.

C (8, 4) =

=

= 70

Therefore, the number of outcomes with the same number of heads and tails is 70.

**20.** The number of bitstrings of lengths 10 are 2^10 = 1024

a).

The number of bit strings of length 10 and having exactly three 0s

= C (10, 3)

=

= 120

Therefore, the number of bit strings of length 10 and having exactly three 0s is 120.

b).

The number of bit strings of length 10 having more 0s than 1s is same as

The number of bit strings of length 10 having 1s less than 0s

So, number of bit strings of length 10 having 1s less than 0s are

= C (10, 0) + C (10, 1) + C (10, 2) + C (10, 3) + C (10, 4)

= 1 + 10 + + +

= 1 + 10 + 45 + 120 + 210

= 386

The number of bit strings of length 10 having more 0s than 1s is 386.

1. **21.**

**a)**

If *BCD* is to be a substring, then we can think of that block of letters as one super letter, and the problem

is to count permutations of five items the letters *A, E, F,* and *G,* and the super letter *BCD.* Therefore

the answer is *P (5,* 5) = 5! = 120.

**b)**

Reasoning as in part (a), we see that the answer is P (4,4) = 4! = 24.

c) As in part (a), we glue *BA* into one item and glue *GF* into one item. Therefore, we need to permute five

items, and there are *P (5,* 5) = 5! = 120 ways to do it.

**d)**

This is similar to part (c). Glue *ABC* into one item and glue *DE* into one item, producing four items,

so, the answer is P (4,4) = 4! = 24.

e) If both *ABC* and *CDE* are substrings, then *ABCDE* has to be a substring. So, we are really just

permuting three items: *ABCDE, F,* and *G.* Therefore, the answer is *P (3,3)* = 3! = 6.

**f)**

There are no permutations with both of these substrings, since *B* cannot be followed by both *A* and *E* at

the same time.

1. **31.**

We need to be careful here, because strings can have repeated letters.

**a)** We need to choose the position for the vowel. and this can be done in 6 ways. Next, we need to choose

the vowel to use, and this can be done in 5 ways. Each of the other five positions in the string can contain

any of the 21 consonants. So, there are 215 ways to fill the rest of the string. Therefore, the answer is

6 \* 5 \* 21^5 = 122,523,030.

**b)** We need to choose the position for the vowels, and this can be done in C (6, 2) = 15 ways (we need to choose

two positions out of six). We need to choose the two vowels ( 5^2 ways). Each of the other four positions in

the string can contain any of the 21 consonants, so there are 214 ways to fill the rest of the string. Therefore

the answer is

15 \* 5^2 \* 21^4 = 72,930,375.

c) The best way to do this is to count the number of strings with no vowels and subtract this from the total

number of strings. We obtain 266 - 216 = 223,149,655.

**d)** As in part (c), we will do this by subtracting from the total number of strings, the number of strings with

no vowels and the number of strings with one vowel (this latter quantity having been computed in part (a)).

We obtain

26^6 \* 21^6 \* 6 \* 5 \* 21^5 = 223149655 - 122523030 = 100,626,625.